Discussion Problems for Math 180

Thursday, April 16, 2015

Review – take no more than five minutes per question.

1. (a) Use a linear approximation to the function $f(x) = \sqrt{x}$ to estimate $\sqrt{8}$.
   (b) Now use a linear approximation to the function $g(x) = 8/\sqrt{x}$ to estimate $\sqrt{8}$.
   (c) Are your answers overestimates or underestimates? What does this tell us about $\sqrt{8}$?
2. Find $\lim_{x \to 0^+} \cot(x)^{2x}$.
3. If $f''(x) > 0$ on the interval $[a, b]$, which of the following are necessarily true?
   (a) $f(x)$ is positive on $[a, b]$.
   (b) If $a < x < b$ then $f(x) > f(a)$.
   (c) The slope of the tangent line to f at a point $a < x < b$ is increasing.
   (d) f is concave up on $[a, b]$.
   (e) $f(b) > f(a)$.
4. Calculate $\dfrac{d}{dx}\left[2 \ln\left(\dfrac{x}{e^x + 1}\right)\right]$.

This time

5. What is the definition of the definite integral $\displaystyle\int_a^b f(x)\, dx$?
6. Calculate integrals:
   (a) $\displaystyle\int_0^{\pi/4} \sec^2 \theta \, d\theta$
   (b) $\displaystyle\int_{-1}^{1} |x| \, dx$
   (c) $\displaystyle\int_2^4 \frac{2}{x} \, dx$
7. If the area between the parabola $y = 1 - ax^2$ (where $a > 0$ is a constant – see illustration below) and the x-axis is 1, what is a?